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The formation of such a wave is illustrated in Figure b, where a transverse wave with a maximum area of  $S=b \cdot CD$  enters the waveguide at the left.

The rectangular components of this wave in the waveguide have the following amplitudes:

$$E'_y = \frac{1}{2} k \frac{\pi}{a}, \quad H'_x = \frac{\gamma}{2} \frac{\pi}{a}, \quad H'_z = \frac{1}{2} \left( \frac{\pi}{a} \right)^2, \quad (3)$$

and the direction of propagation, before and after reflection from the walls, is characterized by the following equations

$$r_1 = z \sin \varphi - x \cos \varphi, \quad r_2 = z \sin \varphi + x \cos \varphi.$$

It is easily seen that magnetic components are generated and give a resulting transverse component with an amplitude:

$$H'_z = H'_x \sin \varphi + H'_y \cos \varphi = E'_y \quad (4)$$

In real waveguides, the electrical conductivity  $\gamma_1$  of walls is quite large but still finite. The initial absorption (of a wave), in connection with this, is usually characterized by the coefficient of attenuation

$$\rho = \frac{P}{2W}. \quad (5)$$

where  $P$  represents the average losses of power in the walls per unit of axial length of the waveguide, and  $W$  represents the average rate of energy flow in the waveguide, computed without considering attenuation.

$\rho$  is usually determined by approximations ([1] and [3]) which sometimes require cumbersome calculations. Not without interest is the method of determining  $\rho$  through representation of the repeatedly reflected waves.

It follows from Figure b, that the number of reflections experienced by a purely transverse wave from both walls of width  $b$  while moving one centimeter along the axis of the waveguide equals

$$N = \frac{1}{a} \cot \varphi. \quad (6)$$

The coefficient of reflection of the amplitude for a wave with the electrical vector perpendicular to the plane of the incidence equals ([4], p 70)

$$R = -\frac{\sin(\varphi - \varphi_1)}{\sin(\varphi + \varphi_1)}, \quad (7)$$

where  $\varphi_1$  is the angle of refraction, determined by the expression

$$\frac{\sin \varphi}{\sin \varphi_1} = n' = n - jk,$$

Here  $n$  represents the principal index of refraction and  $k$  the coefficient of absorption of the reflecting surface. They are determined by the well-known equations of Drude ([4], p 74).

Since in the case of metals, in the presence of microwaves the displacement current is negligible, i.e., in comparison with the conducting current, the following approximate expressions are accurate enough for our purposes:

$$n \approx k \frac{\sqrt{\epsilon_1}}{f} \gg 1 \quad (8)$$

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where  $Y_1$  is expressed in electromagnetic units. Introducing for the sake of abbreviation the value  $A = \frac{\sin \varphi}{2\pi}$ , we obtain after simple computations:

$$|R| = \frac{\left[ \frac{1}{\cos^2 \varphi} + 4\pi^2(1+4A^2) + 2\pi^2 \cos^2 \varphi (1+4A^2) \right]^{1/2}}{\frac{1}{\cos^2 \varphi} + 2\pi^2 \sqrt{1+4A^2} + \frac{1}{2\pi^2 \cos^2 \varphi} \sqrt{1+4A^2} - \frac{2A^2}{1+4A^2}} \quad (8A)$$

Taking into consideration the strong inequality (8), let us obtain a simpler expression, accurate up to the term  $\pi^2$ :

$$|R| = 1 - \Delta \approx 1 - \frac{\cos^2 \varphi}{A},$$

where  $\Delta$  represents a very small magnitude.

Therefore the average losses in the walls of width  $b$  are determined as follows:

$$P_b = W(1 - |R|^2) \approx 2\Delta NW = \frac{2W}{\pi a} \frac{\cos^2 \varphi}{\sin \varphi} \quad (9)$$

To determine losses in walls of width  $a$ , it is necessary to consider that at point B with coordinate  $x$  (Figure b) the phase difference between the  $x$ -components of the interfering transverse waves equals

$$\delta = \frac{2\pi}{\lambda} \cdot 2x \cos \varphi$$

and therefore the square of the amplitude of the resulting  $x$ -components equals

$$H_{x1}^2 = \frac{1}{2} \left( \frac{\pi}{a} \right)^2 [1 + \cos \delta].$$

The  $x$ -component is similarly derived:

$$\delta_1 = \delta - \pi; H_{x1}^2 = \frac{1}{2} \left( \frac{\pi}{a} \right)^2 [1 - \cos \delta].$$

The square of the amplitude of the resulting field at the surface of the wall is determined obviously by the expression:

$$H_1^2 = H_{x1}^2 + H_{y1}^2.$$

This value can be used for computing the losses according to the method of surface effect ([1], p 104). Let us obtain the average losses in the strip ( $a \times 1$ ) cm<sup>2</sup>:

$$P_a = 2 \cdot \frac{1}{2} \cdot \frac{1}{8\pi} \sqrt{\frac{f}{\gamma_1}} \int_0^a H_1^2 dx = \frac{\pi^3}{4} \frac{1}{a\lambda^2} \sqrt{\frac{f}{\gamma_1}}. \quad (10)$$

The value of the average rate of energy flow entering the waveguide is determined by the following expression:

$$W_{10} = \frac{c}{8\pi} (E_y')^2 b \cdot CD = c \frac{\pi^3}{4} \frac{b}{a} \frac{\sin \varphi}{\lambda^2} \quad (11)$$

Taking into consideration (5), (9) and (10), we determine the coefficient of attenuation of wave  $H_{10}$  in the waveguide as follows:

$$\beta_{10} = \frac{1}{2} \frac{P_a + P_b}{W_{10}} = \frac{\frac{1}{2} + \frac{b}{a} \cos^2 \varphi}{\pi b \sin \varphi} \quad (12)$$

Introducing the quantities  $y = \frac{a}{b} = \frac{f}{f_0}$  (where  $f_0$  represents the limiting frequency for a given waveguide) and  $\rho$  (specific resistance expressed in practical units) we obtain

$$\beta_{10} = 0.129 \frac{\sqrt{\rho}}{a^{3/2}} \frac{\frac{a}{2b} y^2 + 1}{\sqrt{y - y_0^2 - 1}}, \quad (13)$$

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which corresponds with values determined through other methods ([5], equation 21.26 and [6], equation 6.146).

It is necessary to point out that, if the coefficient of attenuation  $\beta'_{mn}$  for wave  $H_{mn}$  is calculated according to the method of surface effect and if then it is determined for  $m = 1$ ,  $n = 0$ , then the coefficient of attenuation  $\beta'_{10}$  determined through this method will differ from (13) by the absence of digit 2 in the first term ([1], equation 10.3.3).

A similar error occurs in determining the average current  $W'_{10}$  from current  $W_{mn}$  ([1], equation 10.1.5); this value is found to be twice as small as the real value.

The reason for these differences is that integration with respect to  $y$  in the case of a  $H_{10}$  wave introduces the factor  $b$  and in the case of  $H_{mn}$  wave, when the square of the sine or cosine is integrated, it introduces the factor  $b/2$ .

Therefore, the degenerate  $H_{10}$  wave cannot be regarded as the limiting case of the  $H_{mn}$  wave; of course, this observation is correct also in the case of the  $H_{01}$  wave.

In conclusion, let us point out that this concept of the repeatedly reflecting transverse wave makes it possible to solve in a simple equation the problem of the rapid change of phase in the reflection of a wave from an ideal conductor, in the explanation of which the author was originally vague ([7]).

Actually, from Figure 1 it follows that the transmission of energy by the purely transverse wave (velocity  $c$ ) along path 2BD is equivalent to the transfer of energy along the axis of the wave guide for a distance  $h$  (with a group velocity  $u$ ). In this case, obviously, it follows that

$$\frac{u}{c} = \frac{h}{2BD} = \sin \varphi < 1. \quad (14)$$

In estimating phase velocity  $v$ , which according to (14) must exceed  $c$ , it is necessary to consider that a change of phase of a  $H_{10}$  wave, occurring in sector  $h$ , corresponds to a change of phase of the transverse wave in sector 2BD and the rapid change of phase in reflection, giving an additional increment to the course amounting to  $\pm$  half of a wave length, so that the equivalent course covered by a transverse wave is equal to

$$L = 2BD \pm \frac{\lambda}{2} = \frac{c}{v \sin \varphi} \pm \frac{\lambda}{2}.$$

Therefore, the following ratio results:

$$\frac{c}{v} = \frac{L}{h} = \frac{1}{\sin \varphi} [1 \pm \cos^2 \varphi],$$

which can be less than one only when selecting the lower sign of  $\pm$ ; under this condition the correct ratio is obtained ([1], p 69).

$$\frac{c}{v} = \sin \varphi.$$

Consequently, in the process under consideration, it must be assumed that in reflections from metal the wave loss occurs.

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[Figures follow.]

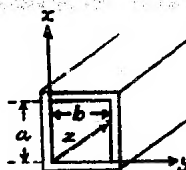


Figure a

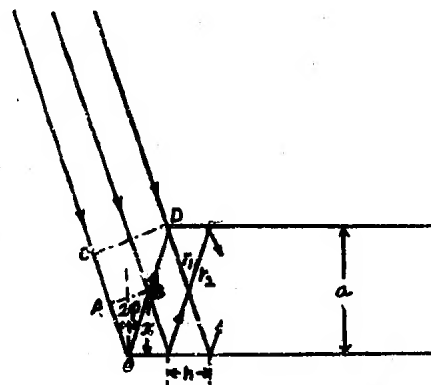


Figure b

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